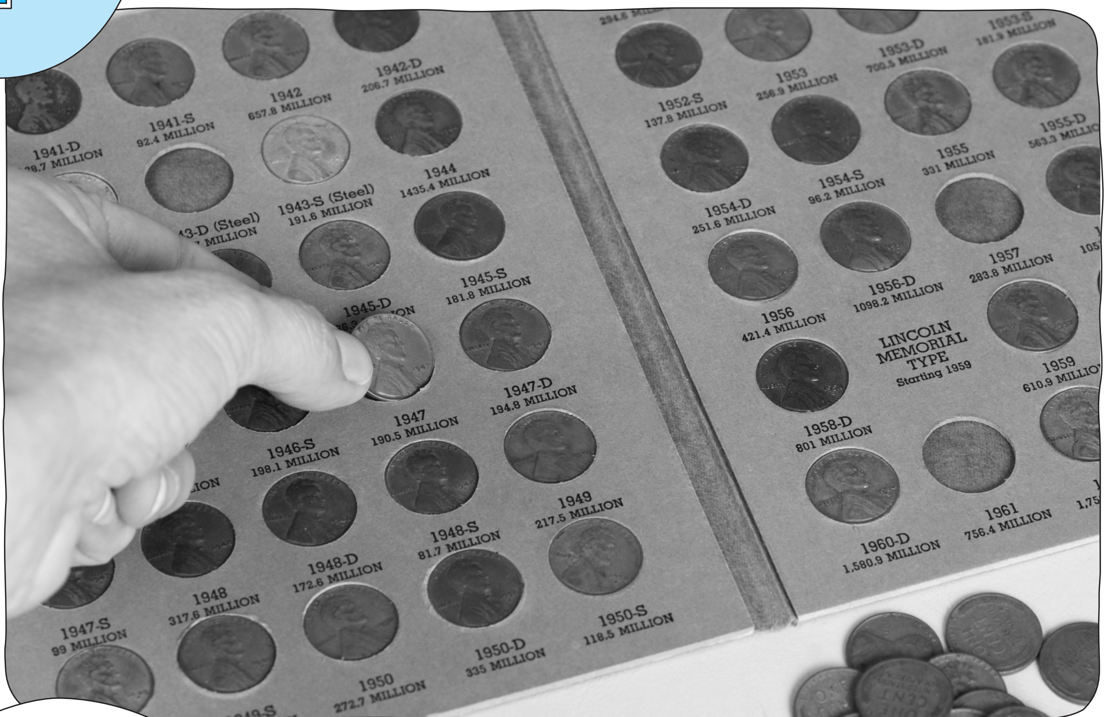
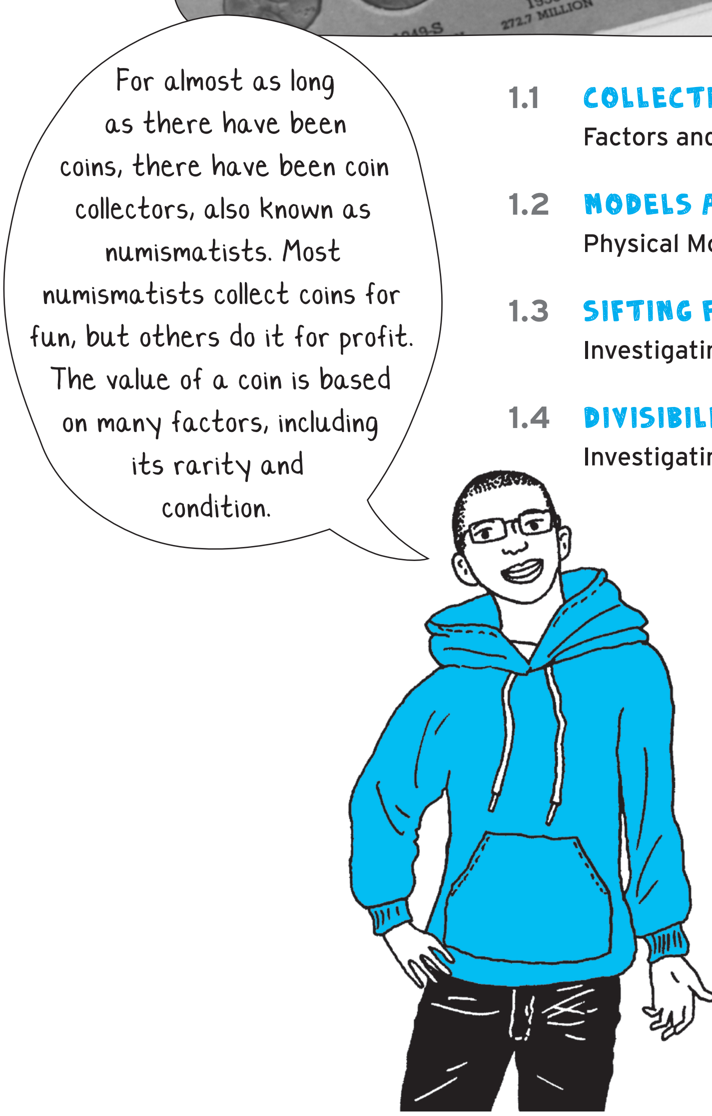
1 FACTORS, MULTIPLES, PRIMES, AND COMPOSITES





1.1 Collection Connections

Factors and Multiples

Learning Goals

In this lesson, you will:

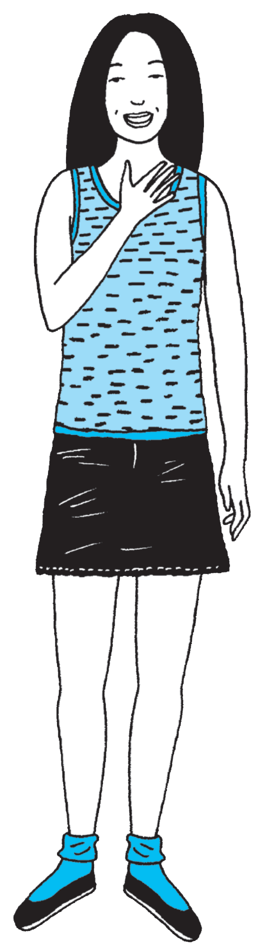
- List factor pairs of numbers.
- Relate factors, multiples, and divisibility.

Key Terms

- array
- factor pair
- factor
- Commutative Property of Multiplication
- distinct factors
- perfect square
- multiple
- divisible

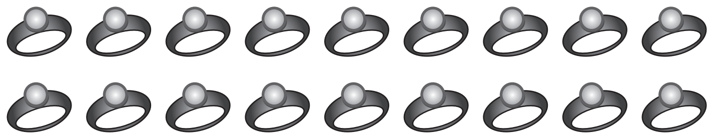
Many people are collectors. They often collect items like stamps, dolls, coins—well, almost anything you can think of! As their collections grow, people often want ways to display their cherished collections. Collectors might also want to group their prized possessions for a variety of reasons and in many different ways.

Do you collect anything? Can you think of some reasons why collectors might want to group their collectables?



Problem 1 Kenya's Rings

Kenya has a collection of 18 rings, and she wants to store them in a box. She uses an *array* that groups her collection into two rows, with each row having nine rings. An **array** is a rectangular arrangement that has an equal number of objects in each row and an equal number of objects in each column.



1. Use tiles to represent Kenya's 18 rings. Arrange the tiles to create other arrays.

a. Describe the arrays you created.

b. What mathematical operation(s) did you think about as you created your arrays?

c. Is there more than one way to make an array with 2 rings in a row or column? Is there more than one way to make an array with 3 rings in a row or column? Explain your reasoning.

d. How do you know when you have determined all of the arrays for a number?

Kenya notices that the number of rings she used in her rows and columns are the same as the *factor pairs* of 18. A **factor pair** is two natural numbers other than zero that are multiplied together to produce another number. Each number in a factor pair is a **factor** of the product.

2. Think about all the factor pairs that could be used to group Kenya's 18 rings.

a. List all the factor pairs.

b. How are the arrays you created in Question 1 related to the factor pairs you just listed?

c. What mathematical operations can you use to determine factor pairs?

d. How will these operations help you determine the factor pairs?

Kenya notices that the array of 2 rows of 9 rings is different from the array of 9 rows of 2 rings. However, when she multiplies the factor pairs for these groupings, the product is the same. In a multiplication sentence, this means $2 \times 9 = 9 \times 2$. The factor pair of 9 and 2 is equal to the factor pair of 2 and 9. This is an example of a very important property called the *Commutative Property of Multiplication.*

The **Commutative Property of Multiplication** states that changing the order of two or more factors does not change the product. For any numbers a and b, $a \times b = b \times a$.

3. Write another example of the Commutative Property of Multiplication using the arrays you created.

4. Will every factor pair in your arrays have an equal pair? Why or why not?

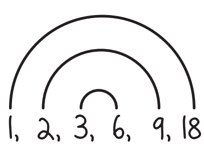
Mr. Rubenstein asked the class to write all of the *distinct factors* that appear in Kenya's arrays for the number 18. **Distinct factors** are factors that appear only once in a list.

5. What are the distinct factors of 18 from Kenya's arrays?

6. How are factors different from factor pairs?

7. Aaron lists all the factor pairs for the number 20. He writes 1 and 20, 20 and 1, 2 and 10, 10 and 2, 4 and 5, 5 and 4. Aaron claims that the number 20 has 12 distinct factors. Abdul does not agree with Aaron's answer. He claims that the number 20 has six distinct factors. Who is correct? Explain how you determined your answer.

8. Marcus noticed that when he listed the distinct factors of a number in order from least to greatest and connected the factor pairs, he could create a rainbow. Marcus drew this picture.



   a. List all of the distinct factors of 20 using Marcus' method.

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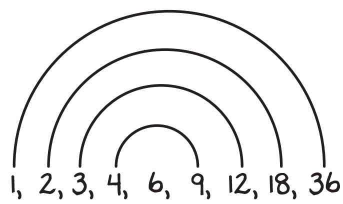
b. List all of the distinct factors of 24. Create a rainbow diagram with the factor pairs.

c. Does every distinct factor of 24 have a partner? Why or why not?

9. Brynn wrote all the distinct factors for 36 in order from least to greatest. She connected the factor pairs and drew this picture.

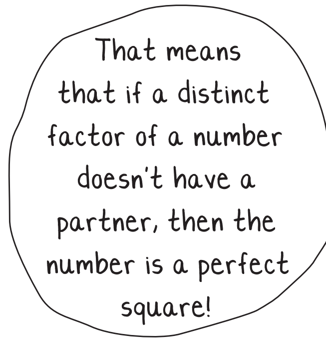


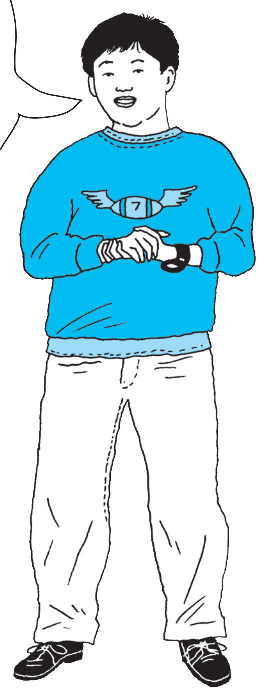
She noticed that 6 did not have a partner.

a. Why do you think 6 does not have a partner?

b. How is this list different from the lists for 20 and 24?

A number that is the product of a distinct factor multiplied by itself is called a **perfect square.** The number 36 is a perfect square because 6 is a distinct factor of 36, and $6 \times 6 = 36$.





10. Choose another perfect square number.

a. Write the number you chose.

b. Order the factors for your perfect square from least to greatest. Then, create a rainbow diagram to show the factor pairs.

c. Which factor multiplies by itself to get the perfect square?

11. Does a perfect square number have an even or odd number of distinct factors? Show how you determined your answer.

12. Show other examples to support your answer.

Problem 2 David's Baseball Cards

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1. David has a collection of 48 baseball cards. He wants to put his cards into different groups.

   a. How can David organize his collection of baseball cards?

   David decides to organize his cards so that each group contains the same number of cards. He can use factors and factor pairs to complete this task.

   b. List all the ways in which David can group his 48 cards into equal groups.

   c. What mathematical operations did you use to determine the groups?

   d. How are David's groups related to the factors of 48?

   e. How can David arrange his cards into equal groups if he has 24 cards?

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2. You previously determined the distinct factors for Kenya's 18 rings and David's 48 baseball cards.

   a. Explain how you determined the distinct factors for each of these numbers.

   b. Exchange the steps of your method for determining distinct factors with your partner. If your partner follows your method, can your partner determine the distinct factors of 28? If you follow your partner's method, will you be able to determine the distinct factors of 28?

   c. Were your methods similar? Did you and your partner determine the same list of distinct factors?

3. Determine the distinct factors of each.

   a. 11

   b. 16

   c. 63

   d. 72

   e. Explain how you determined the distinct factors of 72.

4. Look back at each list of distinct factors in Question 3. What factor is common to all the lists?

5. What did the number 1 represent when you were creating arrays for Kenya's ring collection and the different groupings for David's baseball collection?

6. Do you think 1 is a factor of every number? Explain why or why not.

7. Describe the numbers that will always have 2 as a factor. Explain your reasoning.

8. Describe the numbers that will never have 2 as a factor. Explain your reasoning.

Problem 3 Exploring Multiples

1. Buy Rite Produce and Vitamins Store sells juice cartons. Each carton contains 12 cans of orange juice.
   a. If you buy 1 carton, how many cans of juice will you have?

   b. If you buy 2 cartons, how many cans of juice will you have?

   c. If you buy 7 cartons, how many cans of juice will you have?

2. How did you determine how many cans of juice you would have if you bought one carton of juice? Two cartons of juice? Seven cartons of juice?

3. How does the number 12 relate to the total number of cans of juice? Explain your reasoning.

When you multiply 12 by any other number, you get a *multiple* of 12. A **multiple** is the product of a given whole number and another whole number.

4. List three multiples of 12.

5. If you buy full juice cartons at Buy Rite, can you buy exactly 54 cans? Explain why or why not.

6. If you need to purchase 192 juice cans, how many cartons must you buy? Explain your reasoning.

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Talk the Talk

In mathematics, there are many ways to show the relationship between a number and one of its factors. For instance, you can say that 5 is a factor of 35, or you can say that 35 is a multiple of 5. You can also say that 35 is *divisible* by 5. One number is **divisible** by a second number when the second number divides "evenly" into the first number.

1. Explain why 35 is a multiple of 5.

2. Explain why 35 is divisible by 5.

3. How are multiplication and division related? Give examples to explain your answer.

Be prepared to share your solutions and methods.

1.2 Models and More

Physical Models of Factors and Multiples

Learning Goals

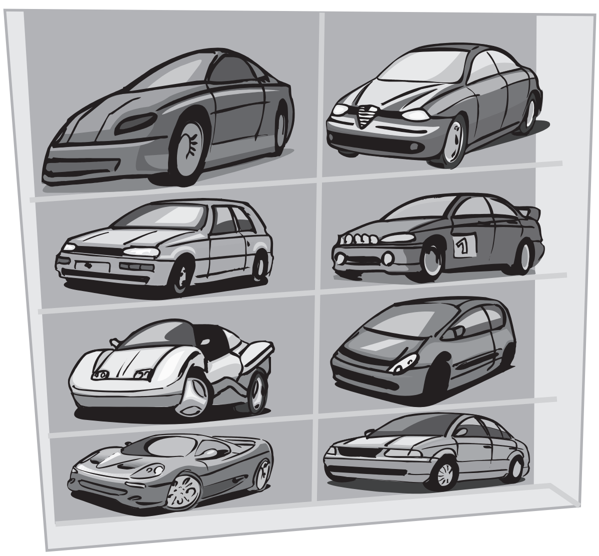
In this lesson, you will:

- Determine factor pairs using arrays and area models.
- Classify numbers using Venn diagrams.

Key Terms

- area model
- set
- Venn diagram

Do you ever wonder how the new cars you see on the road or in car dealership windows are designed and made? Artist sketches and models play a huge part in the process of taking a designer's idea to the cars you see today. Can you think of other items that require artist sketches and models during the creation process?



Problem 1 Using Arrays to Plan Displays

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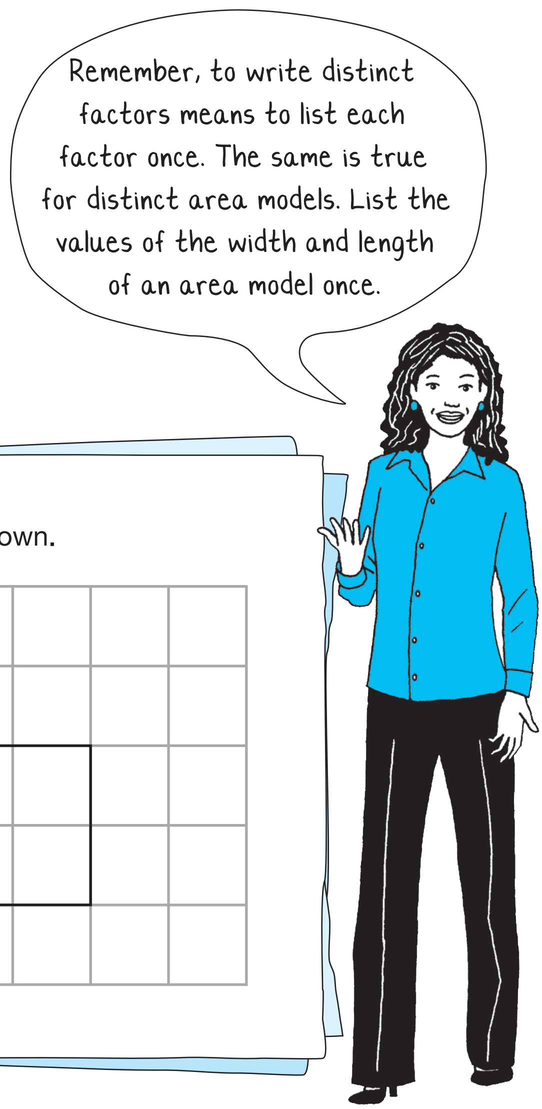
Speedy Builders specializes in creating shelves, desks, and cases for collectible items. Speedy Builders wants to make rectangular cases with individual sections to display model cars. They want to design different-sized cases to display various numbers of cars. Before they build the cases, designers draw arrays on grid paper to make sure mistakes are not made.

Think about a case that holds exactly 12 model cars, with one car in each slot.

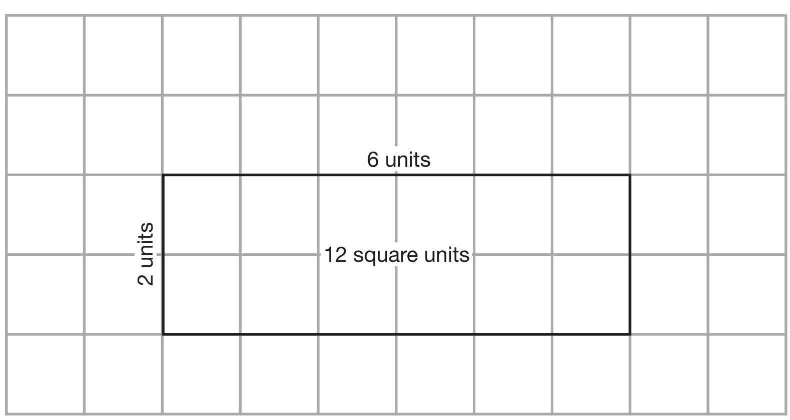
1. What arrays could be drawn?

2. What is the relationship between the arrays drawn and the factor pairs of 12?

You can draw rectangles on grid paper to learn more about factors. Think of factor pairs as the dimensions of any rectangle using an *area model*. An **area model** for multiplication is a pictorial way of representing multiplication. In the area model, the rectangle's length and width represent factors, while the rectangle's area represents the product.



An area model for one factor pair of 12 is shown.



3. Draw all of the possible distinct area models for each number from 1 through 30. You may want to use tiles before you draw your models on grid paper. Label all of the dimensions of your models.

Distinct Models for Numbers 1 through 5

Distinct Models for Numbers 6 through 10

Distinct Models for Numbers 11 through 15

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Distinct Models for Numbers 16 through 20

Distinct Models for Numbers 21 through 25

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Distinct Models for Numbers 26 through 30

a. Which numbers have only one distinct area model? What do you know about these numbers?

b. Which numbers have a perfect square as one of their distinct area models? Explain your reasoning.

c. How are the factors of a number related to the dimensions of a distinct area model?

d. Recall that Speedy Builders is trying to decide which case they should design to hold 12 cars. Which display(s) would you recommend that Speedy Builders create? Explain your reasoning.

Problem 2 Arranging More than Music Notes

Mr. Jeffries, the music teacher at Harrison Middle School, has 36 students in his chorus. He wants to place the desks in his room in equal rows. He has a very large room, but he can't decide which arrangement he should use.

1. What are all the possible ways in which Mr. Jeffries can arrange the chairs in equal rows?

2. List all of the distinct factors for 36. Then, state the number of distinct factors of 36.

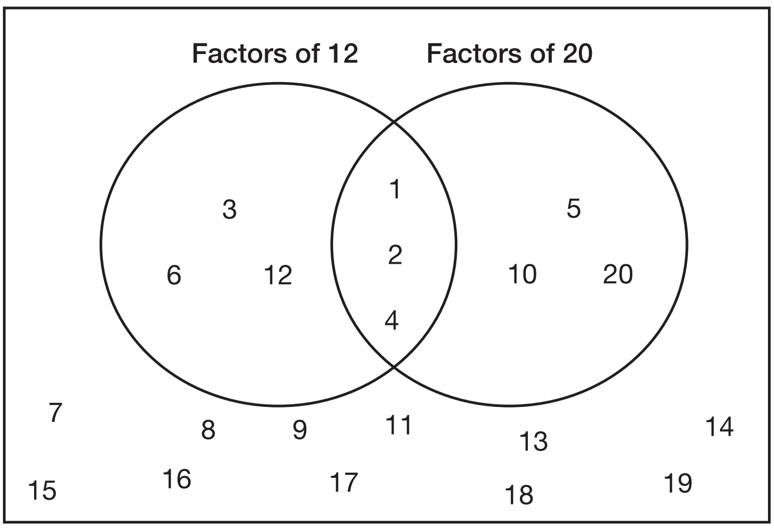
3. What do you know about a number that has an odd amount of distinct factors? Explain your reasoning.

4. What recommendation would you make to Mr. Jeffries about his room arrangement? Explain why you selected this arrangement.

Problem 3 Venn Diagrams

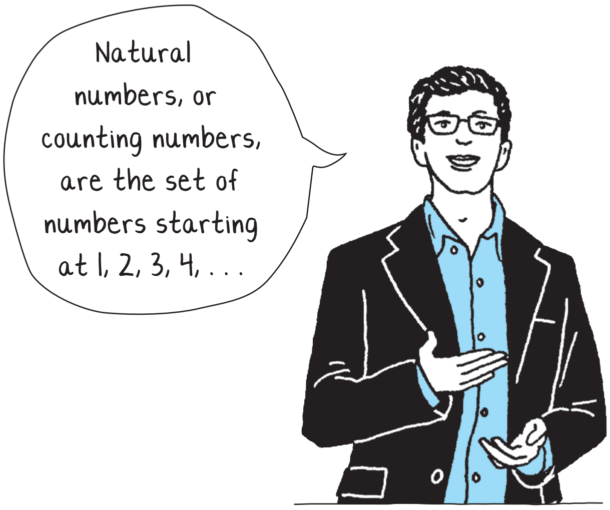
You have created models to show relationships between numbers. Sometimes when numbers are related in some way to each other, the numbers are called a *set*. A **set** is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common. Another model you can use to show relationships among numbers is a *Venn diagram*, named after British logician and philosopher John Venn. A **Venn diagram** is a picture that illustrates the relationships between two or more sets. Venn diagrams use circles to help you put numbers into common groups.

The example shown represents the natural numbers between 1 and 20 and the relationship between those numbers that are factors of 12, factors of 20, or factors of neither 12 nor 20.



The numbers that are factors of 12 are in the region labeled "Factors of 12." The numbers that are factors of 20 are in the region labeled "Factors of 20." The numbers that are factors of both 12 and 20 are in the center where the circles overlap, or intersect.

Notice that there is no label for those numbers that are factors of neither 12 nor 20. If numbers do not fit into either category, they are outside of the circles but within the rectangle.



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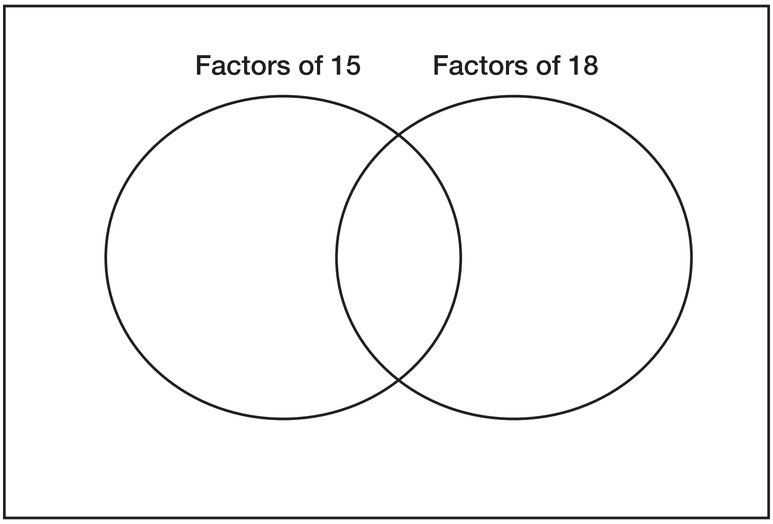
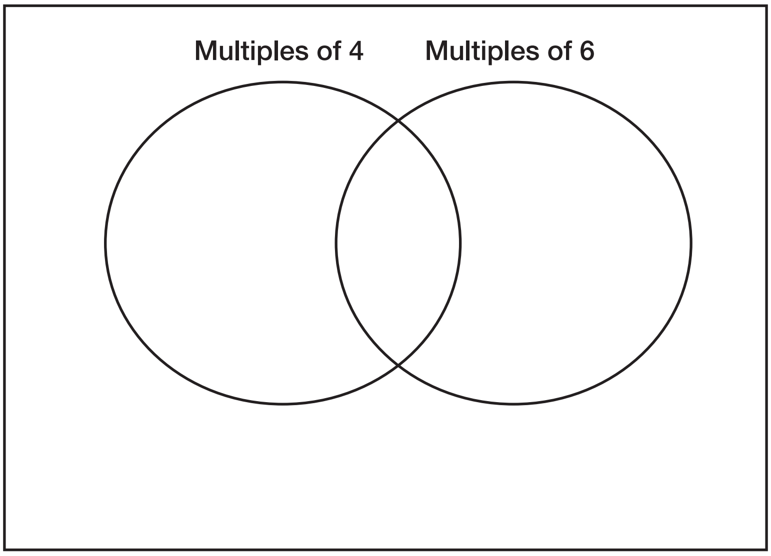
1. Complete the Venn diagram to show the factors of 15 and 18. Use the natural numbers between 1 and 20.



a. What do the numbers in the overlapping, or intersecting, region represent?

b. What do the numbers outside of the circles represent?

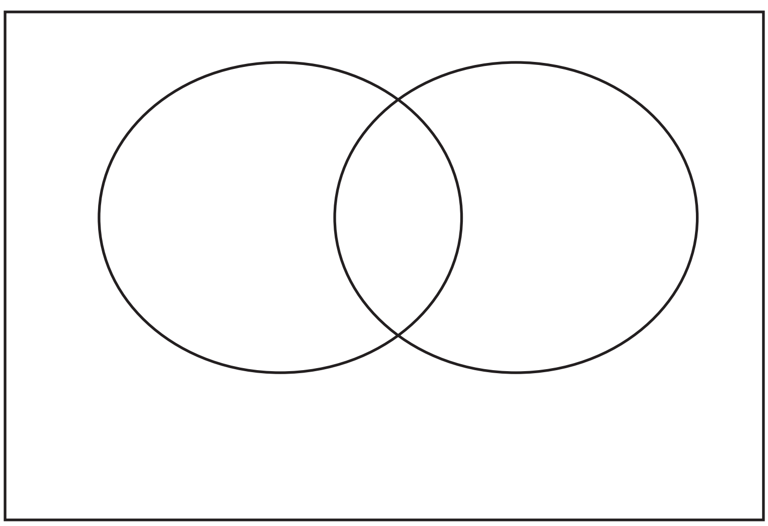
2. Complete the Venn diagram to show the multiples of 4 and 6. Use the natural numbers between 1 and 30.



a. What do the numbers in the overlapping, or intersecting, region represent?

b. What do the numbers outside of the circles represent?

3. Create a Venn diagram that shows the factors of 24 and 40.



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4. Answer each question using the Venn diagram you created.

a. List all the distinct factors of 24.

b. List all the distinct factors of 40.

c. List the common factors of 24 and 40, if possible.

5. How can Venn diagrams help you organize information?

Be prepared to share your solutions and methods.

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1.3 Sifting for Prime Numbers

Investigating Prime and Composite Numbers

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Learning Goals

In this lesson, you will:

- Distinguish between prime and composite numbers.
- Identify and use the multiplicative identity.

Key Terms

- prime numbers
- composite numbers
- multiplicative identity

You may have heard that during the time of Columbus, many people believed that the Earth was flat. However, some historians think that this is a misconception.

These historians do make a good point. Greek mathematician, geographer, and astronomer Eratosthenes (pronounced Er-uh-TOSS-thuh-neez) has been credited as the first person to calculate the circumference of the Earth. He made this calculation roughly around 240 BCE — thousands of years before Columbus' time! If you remember, circumference is the distance around a circle. Therefore, even back then, people assumed that the Earth was round.

So, where do you think this misconception that many people in the 1400s believed in a flat Earth came from?

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Problem 1 The Game

A sieve is an old tool that is used to separate small particles from larger particles and is usually a box with a screen for a bottom that allows the smaller pieces to fall through.

The Sieve of Eratosthenes screens out all of the *composite* numbers and leaves only the *prime numbers*. **Prime numbers** are numbers greater than 1 with exactly two distinct factors, 1 and the number itself. **Composite numbers** are numbers that have more than two distinct factors. You and your partner will use the Sieve of Eratosthenes to determine all of the prime numbers up to 100.

The figure shows the first 100 numbers written in numerical order in an array.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Start by putting a square around the number 1 because it is not a prime or composite number.
2. Circle the number 2 and cross out all of the multiples of 2.
3. Circle the next number after 2 that is not crossed out. Then cross out all the multiples of that number that are not already crossed out.
4. Continue in this fashion until you come to the first number greater than 10 that is not crossed out. All of the remaining numbers have “fallen through the sieve” and are prime numbers.
5. List all of the prime numbers up to 100.

6. How many of the prime numbers are even?

7. Is it possible that there are even prime numbers greater than 100? Explain your reasoning.

8. Why did you stop at 10? How do you know that any remaining number less than 100 must be a prime number?

9. Are all odd numbers prime? Explain your reasoning.

10. Recall the distinct area models you created for Speedy Builders in the previous lesson. Identify the prime numbers. How are all the area models of prime numbers similar?

Talk the Talk

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- The number 1 is neither prime nor composite.
- The number 1 is a factor of every number.

The number 1 is called the *multiplicative identity*. The **multiplicative identity** is the number 1. When it is multiplied by a second number, the product is the second number. An example is $1 \times 5 = 5$.

1. Explain why the number 1 is neither prime nor composite.

2. State the characteristics prime numbers share.

3. State the characteristics composite numbers share.

Be prepared to share your solutions and methods.

1.4 Divisibility Rules!

Investigating Divisibility Rules

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Learning Goals

In this lesson, you will:

- Formulate divisibility rules based on patterns seen in factors.
- Use factors to help you develop divisibility rules.

Key Term

- divisibility rules

Understanding relationships between numbers can save you time when making calculations. Previously, you worked with factors and multiples of various numbers, and you determined which numbers are prime and composite by using the Sieve of Eratosthenes. By doing so, you determined what natural numbers are divisible by other natural numbers.

In this lesson, you will consider patterns for numbers that are divisible by 2, 3, 4, 5, 6, 9, and 10. What type of patterns do you think exist between these numbers? Why do you think 1 is not a part of this list?

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Problem 1 Exploring Two, Five, and Ten

1. List 10 multiples for each number.

Multiples of 2:

Multiples of 5:

Multiples of 10:

2. What do you notice?

Divisibility rules are tests for determining whether one whole number is divisible by another. A divisibility rule must work for every number.

3. Write a divisibility rule for 2, 5, and 10. Then, show an example that follows your rule.

A natural number is divisible by	...if	Example
2		
5		
10		

Problem 2 Exploring Three and Six

Each number shown in the table is divisible by 3.

Number	Divisible by 2	Divisible by 3	Divisible by 5	Divisible by 10
300		✓		
1071		✓		
882		✓		
1230		✓		
285		✓		
3762		✓		
42		✓		
2784		✓		
3582		✓		
111		✓		

1. Place a check in the appropriate column for each number that is divisible by 2, 5, or 10.

2. Analyze each number that is divisible by 3. Then, write a rule in the table shown to indicate when a number is divisible by 3. (Hint: Consider the sum of the digits of the number.)

A number is divisible by	...if	Example
3		

3. Circle numbers you think are divisible by 6 in the table you completed in Question 1. Explain your reasoning.

4. Analyze each number you circled that you think is divisible by 6. Write a rule to indicate when a number is divisible by 6 in the table shown.

A number is divisible by	...if	Example
6		

5. Test the divisibility rules you wrote to indicate if a number is divisible by 3 or 6 by writing several three- or four-digit numbers that you think are divisible by 3 or 6. Then, use your calculator to determine if the numbers you wrote are divisible by 3 or 6.

Problem 3 Exploring Nine

1. Place a check in the appropriate column for each number that is divisible by 2, 3, 5, 6, or 10. The column for Divisible by 9 is completed for you.

Number	Divisible by 2	Divisible by 3	Divisible by 5	Divisible by 6	Divisible by 9	Divisible by 10
3240					✓	
1458					✓	
18,225					✓	
2025					✓	
33						
7878						
3477						
2565					✓	
285						
600						

2. Analyze the numbers shown in the list. Write a rule to indicate when a number is divisible by 9. (Hint: Use the same clue you were given when exploring the divisibility rule for 3.)

A number is divisible by	...if	Example
9		

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3. Test the divisibility rule you wrote to indicate if a number is divisible by 9 by writing several four- or five-digit numbers that you think are divisible by 9. Then, use your calculator to determine if the numbers you wrote are divisible by 9.

Problem 4 Exploring Four

Each number listed in the table is divisible by 4.

Numbers Divisible by 4	
116	35,660
1436	18,356
228	300,412
2524	59,140
41,032	79,424

1. What pattern do you notice about each number? (Hint: Look at the number formed by the last two digits in each number.)

2. Write a rule to tell when a number is divisible by 4.

A number is divisible by	...if	Example
4		

3. Test the divisibility rule you wrote to indicate if a number is divisible by 4 by writing several five-digit numbers that you think are divisible by 4. Then, use your calculator to determine if the numbers you wrote are divisible by 4.

Problem 5 It's a Mystery

1. Determine if each number is divisible by 3 using your divisibility rule. Explain your reasoning.

   a. 597

   b. 2109

   c. 83,594

2. Determine if each number is divisible by 9 using your divisibility rule. Explain your reasoning.

   a. 748

   b. 5814

   c. 43,695

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3. Fill in the missing digit for each number to make the sentence true.

   a. The number 10,5__2 is divisible by 6.

   b. The number 505__ is divisible by 4.

   c. The number 133,0__5 is divisible by 9.

4. Rasheed is thinking of a mystery number. Use the following clues to determine his number. Explain how you used each clue to determine Rasheed's number.

   Clue 1: My number is a two-digit number.

   Clue 2: My number is a multiple of 5, but does not end in a 5.

   Clue 3: My number is less than 60.

   Clue 4: My number is divisible by 3.

5. Think of your own mystery number, and create clues using what you know about factors, multiples, and the divisibility rules. Give your clues to your partner. See if your partner can determine your mystery number!

Talk the Talk

Divisibility rules are tests for determining whether one number is divisible by another number.

A number is divisible by:

- 2 if the number is even.
- 3 if the sum of the digits is divisible by 3.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if the number ends in a 0 or a 5.
- 6 if the number is divisible by both 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the last digit is 0.

1. Determine if each number is divisible by 8 using the divisibility rule.

a. 75,024

b. 1466

c. 19,729

d. 1968

Be prepared to share your solutions and methods.

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Chapter 1 Summary

Key Terms

- array (1.1)
- factor pairs (1.1)
- factor (1.1)
- distinct factors (1.1)
- perfect square (1.1)
- multiple (1.1)
- divisible (1.1)
- area model (1.2)
- Venn diagram (1.2)
- set (1.2)
- prime number (1.3)
- composite number (1.3)
- multiplicative identity (1.3)
- divisibility rules (1.4)

Property

- Commutative Property of Multiplication (1.1)

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1.1 Listing Factors and Multiples

A factor pair is two natural numbers other than zero that are multiplied together to produce another number. Each number in a factor pair is a factor of the product.

A factor of a number divides evenly into the number. A multiple of a number is the product of the number and any other number.

Example

The factors of 12 and the first five multiples of 12 are shown.

Factors: 1, 2, 3, 4, 6, and 12

Multiples: 12, 24, 36, 48, and 60

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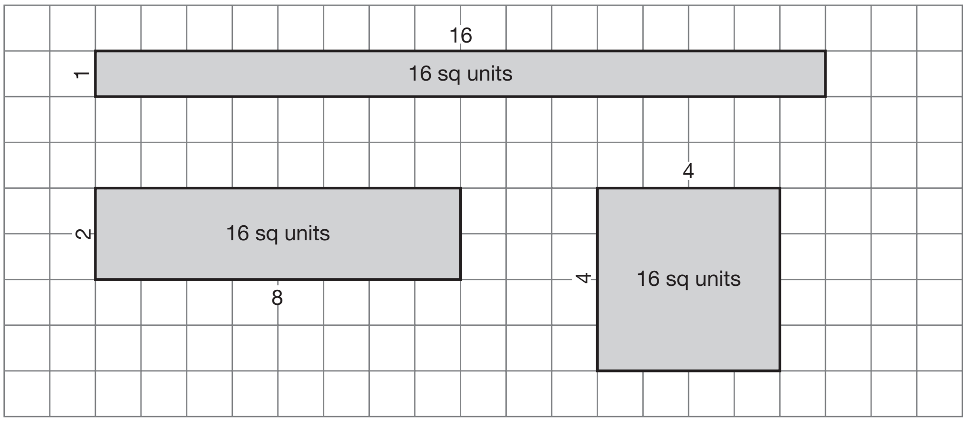
1.2 Determining Factors Using Models

Rectangular arrays and area models can be used to illustrate multiplication and determine factors.

Example

The distinct area models for 16 are shown.



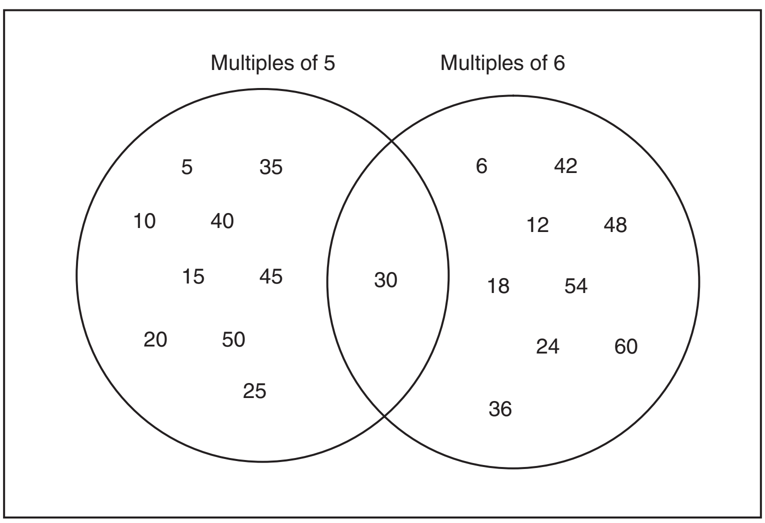
Using the area models, the distinct factors of 16 are 1, 2, 4, 8, and 16.

1.2 Classifying Numbers Using Venn Diagrams

A Venn diagram uses circles to represent the relationship between two or more sets.

Example

The Venn diagram shows the first 10 multiples of 5 and 6.



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1.3 Distinguishing between Prime and Composite Numbers

Prime numbers have exactly two distinct factors, 1 and the number itself. Composite numbers have more than two distinct factors.

Example

Circled numbers are prime numbers. Numbers with a square are composite numbers. The number 1 is neither prime nor composite.

[68] (73) (29) 1 [81] [95] (5) [54] [27] [42] [36] (61)

1.3 Identifying the Multiplicative Identity

The multiplicative identity is the number 1. When it is multiplied by a second number, the product is the second number.

Example

In the number sentence $10 \times \underline{\ ?\ } = 10$, 1 makes the equation true.

The number 1 makes that statement true because the product of 10 and 1 is 10.

1

1.4 Applying Divisibility Rules

A number is divisible by:

- 2 if the number is even.
- 3 if the sum of the digits is divisible by 3.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if the number ends in a 0 or 5.
- 6 if the number is divisible by both 2 and 3.
- 8 if the number formed by the last three digits is divisible by 8.
- 9 if the sum of the digits is divisible by 9.
- 10 if the last digit is 0.

Example

A four-digit number that is divisible by 2, 3, 4, 5, 6, 8, 9, and 10 must have these properties:

- The number must end in 0 to be divisible by 2, 5, and 10.
- The last two digits of the number must be divisible by 4.
- The last three digits must be divisible by 8.
- The sum of all of the digits must be divisible by both 3 and 9.
- The number will automatically be divisible by 6 because it is divisible by both 2 and 3.

The number 6120 is divisible by 2, 3, 4, 5, 6, 8, 9, and 10 using the divisibility rules.